Nowhere dense set and frist category set

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In a topology space, a set E is called *nowhere dense set* if

$$\overline{E}^o = \emptyset$$

and A is call *first category set.* if A is a countable union of nowhere dense set.

According to above definition, we know that a nowhere dense set must be a first category set. then ,in what condition can a first category set become a nowhere dense set?

Baire category theorm say that ,In a complete metric space, a first category set has no inner point.I express it in a mathematical way:

If $\{E_n\}_{n=1}^{n=\infty}$ is a union of nowhere dense set then

$$\left(\bigcup_{n=1}^{n=\infty} E_n\right)^o = \emptyset$$

Our purpose is to find a condition such that

$$\overline{\bigcup_{n=1}^{n=\infty} E_n}^o = \emptyset$$

but In a separable space and every point in the space is nowhere dense, it must contradictary to our purpose.So we can only find our condition in a unseparable space.

I suspect that a first category set in a unseparable space must be a nowhere dense set, but now,I cann't thought how to prove it.